Are characteristic interactions important to the cross-section of expected returns?

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Abstract

Characteristic interactions play an important role in describing the cross-section of expected returns. I use a Fama-Macbeth regression modified to accommodate more variables than observations to study the cross-sectional relationship between characteristic interactions and expected returns. The modified Fama-Macbeth regression uses a form of dimension reduction called an *envelope*, which does not require variable selection or slope regularization. I use the method to estimate the information in 3,655 characteristic interactions about the cross-section of expected returns. About 100 interactions have incremental information about expected returns. Standard long-short portfolios constructed from interaction-based estimates of expected returns have significant risk-adjusted returns compared to standard factor models.

Keywords: Cross-sectional expected stock returns, firm characteristics, statistical envelopes, dimension reduction, big data

JEL Codes: C53, C55, C58, G12, G14, G17

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1 Introduction

Fama and MacBeth (1973) regressions are the standard method for estimating cross-sectional models of expected returns with firm characteristics. Fama and French (1992) uses Fama–Macbeth regressions to show a firm's price-to-earnings ratio does not provide independent information about expected returns after controlling for firm size and book-to-market ratio. Haugen et al. (1996) uses Fama–Macbeth regressions to estimate stocks' expected returns with 41 characteristics[1]. Lewellen (2015) compares realized returns with Fama–Macbeth regression estimates of expected returns. Green et al. (2017) estimates expected returns with Fama–Macbeth regressions and a panel of 94 characteristics weighted to reduce micro-cap stocks' effect on the expected return estimates. Han et al. (2019) estimates expected returns using several Fama–Macbeth regression variations and roughly 300 firm characteristics.

Fama–Macbeth regressions are useful for panels with a moderate number of characteristics but are less well-suited for estimating expected returns with many characteristics for four reasons. First, the standard Fama–Macbeth regression is not defined for panels where one or more cross-sections have fewer observations than characteristics. The standard Fama–Macbeth procedure estimates each cross-section's slopes with the ordinary least squares regression model, which applies only to estimation problems where the number of observations exceeds the number of covariates. Second, high cross-characteristic correlations can make slope estimates imprecise because of a collinearity problem among characteristics. For instance, Green et al. (2017) drops 8% of their initial characteristic panel to mitigate the effects of multicollinearity when running Fama–Macbeth regressions of stock returns on firm characteristics. Dropping highly correlated characteristics improves remaining characteristics' precision but attenuates the relationship between expected returns and characteristics (Lubotsky and Wittenberg, 2006). Third, running cross-sectional regressions with a relatively large number of covariates compared to observations produces slope estimates that are overfitted and weakly correlated with future returns (Han et al., 2019; Freyberger et al., 2020). Last, Fama–Macbeth regressions with too many characteristics are likely less efficient than constrained regression approaches that capitalize on the overlap in

[1] Haugen et al. (1996) is perhaps the first to use Fama–Macbeth regressions to estimate expected returns with such a large set of characteristics.

characteristics' information content about expected returns (Light et al., 2017).

I propose a modified Fama–Macbeth regression for estimating expected returns with many characteristics, including settings with more characteristics than cross-sectional observations. The modified Fama–Macbeth procedure uses a constrained variant of ordinary least squares to run cross-sectional regressions instead of ordinary least squares. The constrained variant of ordinary least squares is the Predictor Envelope Regression (PER) model from Cook et al. (2013). The PER model is the same as ordinary least squares except for the addition of constraints collectively called an envelope, a form of targeted dimension reduction. The envelope constraint lets the PER model consistently estimate both expected returns and slopes for cases with more characteristics than observations (Cook et al., 2019). Practically, the PER model assumes a cross-section's characteristics can be repackaged into a smaller number of psuedo-characteristics that preserve the original characteristics' information about expected returns.

I construct out-of-sample expected return estimates with 3,655 characteristic interactions and the modified Fama-Macbeth regression model to study the relationship between characteristic interactions and the cross-section of expected returns. Out-of-sample expected return estimates are a direct means of investigating the aggregate relationship between many variables and the cross-section of expected returns (Haugen et al., 1996). Each month, I use a ten-year window and the modified Fama–Macbeth regression to estimate the average relationship between characteristic interactions and expected returns. Next, I use the model to predict returns over the following month. The standard Fama–Macbeth regression cannot be used for this estimation problem because the typical cross-section of US stocks without microcaps has about 2,000 stock return observations over the paper's sample period. The 3,655 characteristic interactions are produced by a panel containing 85 standard firm characteristics. The characteristics are based on Green et al. (2017) constructions and include well-known variables like market beta, size, accruals, and momentum.

I show five main results. First, I show characteristic interactions are associated with statistically and economically significant information about the cross-section of expected returns using portfolios built from a decile sort on stocks' interaction-based expected return estimates. For the equal-weight decile portfolios, average returns smoothly increase from the low portfolio, which holds stocks with the lowest expected return estimates, to the high portfolio, which contains stocks with the greatest expected return estimates. The same pattern of increasing average returns holds for the value-weighted decile portfolios. A standard equal-weight long-short portfolio that longs stocks with high predicted returns and shorts stocks with low predicted returns has a positive and significant average monthly return of 3.86% and an annual Sharpe ratio of 3.90. For reference, a comparable long-short portfolio built with expected return estimated based on the original 85 characteristics has an average monthly return of 3.65% and an annual Sharpe ratio of 2.42.

Second, I show that standard multifactor models of stock returns do not explain the average excess returns of long-short interaction portfolios. I include results for the Carhart (1997) four-factor model, Hou et al. (2015) q-factor model, Fama and French (2015) five-factor model, and the Fama and French (2015) five-factor model plus the "winners minus losers" momentum factor. The interaction long-short portfolios have positive and significant risk-adjusted returns for all of the factor models. The interaction long-short portfolios also have positive and significant risk-adjusted returns when I include long-short portfolios for the characteristic-based decile sorts in the multifactor regressions as additional factors.

Third, I use standard Fama–Macbeth regressions to show both characteristic interactions and characteristics contain incremental information about the cross-section of expected returns absent from the other collection of variables. Specifically, I run Fama–Macbeth regressions of stock returns on the interaction-based expected return estimates, characteristic-based expected return estimates, and combinations of the two expected return estimates. Slopes from Fama-Macbeth regressions of stock returns on interaction-based expected return estimates are positive for all stock samples. When I run Fama–Macbeth regressions of stock returns on both interaction- and characteristic-based expected return estimates, the slope for the interaction-based expected return estimates remains positive and significant. I also use observations from Lewellen (2015) to evaluate the bias of the interaction- and characteristic-based expected return estimates' cross-sectional variance. Both the interaction- and characteristic-based expected return estimates exhibit more cross-sectional variance than realized returns. Combination estimates of expected returns incorporating both the interaction- and characteristic-based expected return estimates have cross-sectional variances much closer to the cross-sectional variance of realized returns.

Fourth, I examine which characteristic interactions contain incremental information about expected returns in the cross-section. I find about 100 interactions have incremental information. Within stocks without microcaps, 154 interactions contain incremental information about expected returns at the 1% significance level. And, within large stocks, 88 interactions have incremental information at the 1% significance level. I show the number of interactions with significant and incremental information about expected returns substantially exceeds the expected number of type-I errors under the reported significance levels' null hypotheses. I also use Wald tests to show most of the 85 characteristics produce at least one interaction with significant and incremental information about expected returns.

Fifth, I show the paper's results are robust to the specification of the PER model's single *tuning* parameter. The tuning parameter is a positive integer that specifies the number of repackaged variables the model uses to represent cross-sectional information in the original variables about expected returns (Cook et al., 2013). If the chosen value is too small, then the predictor model's slopes and expected return estimates will omit information in the original variables about expected returns and underestimate the relation between expected returns and characteristics. If the chosen value is too large, the model's slopes will be less precise because the slope estimates depend on some variation in characteristics uncorrelated with the cross-section of expected returns. I use a standard, sequential F-test from Osten (1988) to estimate the tuning parameter's proper value for the paper's main results.

This paper provides a new method for estimating and testing relationships between expected returns and large panels of firm characteristics, including settings where characteristics are as numerous or more numerous than cross-sectional observations. The modified Fama–Macbeth regression is a tool well-suited to wrangling the cross section's predictor zoo while preserving the standard Fama–Macbeth regression specification. Developing methods for estimating expected returns from the available predictor zoo is useful for several lines of financial research. Expected returns can be used to improve portfolio optimization (Treynor and Black, 1973), estimate firms' cost of capital, generate benchmark portfolios for investment managers (Chan et al., 2009), construct more powerful basis assets for asset pricing tests (Haugen et al., 1996), and help the direct study of expected returns (Lewellen, 2015).

A large number of variables likely provide incremental information about the cross-section (Kozak et al., 2020). Models of expected returns typically give central roles to a few intuitive but unobserved variables like expected future profitability and firm quality. For example, expected future profits are not observable, and a variety of firm characteristics can provide some information on them (Fama and French, 2000). Likewise, firm quality is a combination of four unobserved concepts, profitability, growth, safety, and payouts. Each of these four unobserved concepts has many potential proxy variables. Kozak et al. (2020) find that stochastic discount factors (SDFs) constructed from many characteristics perform much better out of sample than SDFs built with only a few firm characteristics. Kozak et al. (2020) also find that characteristics provide information about a relatively small number of common factors in the cross-section. Light et al. (2017) use partial least squares and find that cross-sectional expected return estimates constructed with many characteristics perform well out-of-sample and capture information about expected returns unexplained by factor models built with a few firm characteristics. Light et al. (2017) also note that the information contained in their characteristic sample about expected returns can be capture by a few common factors. Han et al. (2019) finds expected return estimates using many characteristics, model averaging, and penalized regression techniques better explain the cross-section of expected returns than cross-section models using a few characteristics.

Statistical envelopes are a form of dimension reduction designed for estimating regression models when covariance between the regression model's variables is well described by a relatively small number of variables built by repackaging the original variables (Cook et al., 2010, 2013). This assumption is consistent with both the conceptual and empirical relationships the existing factor model literature uses to interpret the relationship between the cross-section of expected returns and firm characteristics. Models that link observed characteristics to expected returns via expected earnings, quality, or other unobserved, conceptual variables describe the relationship the PER model assumes between returns and observed characteristics via repackaged variables. Empirically, Light et al. (2017) use partial least squares to repackage several observed characteristics into one expected return characteristic and find that the repackaged characteristic and expected returns are positively correlated. The model averaging and combination expected return forecasts from Han et al. (2019) also indicate extensive characteristic collections can be repackaged into fewer psuedo-characteristics while preserving the original characteristics' information about expected returns. The characteristic-based factor-beta estimates from Kelly et al. (2019) also show repackaging observed characteristics into a smaller number of psuedo-characteristics can preserve the original characteristics' information about expected returns.

This paper also contributes to a growing literature using contemporary statistical and machine learning techniques for high-dimensional empirical asset pricing. Rapach et al. (2013) uses the lasso to select variables for predicting international stock returns. Kelly et al. (2019) build a generalization of PCA which accommodates time-varying factor loadings and uses firm characteristics as proxies for factor loadings. Kozak et al. (2020) uses shrinkage to estimate the stochastic discount factor using many characteristics. Freyberger et al. (2020) estimates a non-parametric model of expected returns using a group lasso to select characteristics. Giglio and Xiu (2019) uses a three-pass regression procedure, PCA, and many portfolios to estimate the stochastic discount factor when some economically relevant factors may be omitted from the SDF's specification. Han et al. (2019) proposes using a combination of model averaging, penalized regressions, and combination forecasts to estimate cross-section models of expected returns from firm characteristics with many characteristics. In contrast, this paper contributes a generalization of the Fama–Macbeth regression appropriate for estimation problems with thousands of firm characteristics by leveraging the that result most characteristics are proxy variables for a small number of latent characteristics that effectively describe the cross-section of expected returns.

I organize the paper as follows. Section 2 describes the modified Fama–Macbeth regression model with envelope-based dimension reduction. Section 3 describes the paper's empirical analysis of the relationship between the cross-section of expected returns and firm characteristics' interactions. Section 4 concludes the paper.

2 Fama–Macbeth Regressions via Envelope Methods

This section presents the paper's methodology. Section 2.1 describes the standard Fama–Macbeth regression using OLS to estimate cross-sectional regression slopes. Section 2.2 introduces the PER model from Cook et al. (2013). Section 2.3 defines the modified Fama–Macbeth regression procedure.

2.1 Standard Fama–Macbeth Regression

A fundamental question for empirical asset pricing is why average returns vary across assets. The Fama–Macbeth regression provides a specification, point estimates, and standard errors appropriate for answering this question. The Fama–Macbeth regression estimates the cross-sectional relationship between stocks' average returns and a collection of explanatory variables using a multiple regression specification. The specification's explanatory variables can be factor betas or firm characteristics depending on the estimation problem. I will work with firm characteristics, which is standard for recent cross-sectional literature using many firm characteristics.

The Fama–Macbeth regression examines how cross-sectional variation in stocks' expected returns are related to a collection of firm characteristics,

$$Er_{i,t} = \sum_{j=1}^{J} b_j x_{i,t,j}, \tag{1}$$

where $Er_{i,t}$ is the expected return of stock i for month t, b_j is the effect of characteristic j on expected returns, and $x_{i,t,j}$ is characteristic j observed for firm i at the beginning of month t. The equation's unknown variables are the b_j slopes.

The Fama–Macbeth regression's slopes, b_j, are estimated in two steps. First, run monthly cross-sectional OLS regressions of stock returns on firm characteristics. The regression for month t is

$$r_{i,t} = a_t + \sum_{j=1}^{J} \hat{b}_{j,t} x_{i,t,j} + e_{i,t}. \tag{2}$$

Second, compute the sample average of the monthly, cross-sectional regression slopes:

$$\hat{b}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{b}_{j,t}. \tag{3}$$

2.2 Predictor Envelope Regression

The specification for a cross-sectional regression of stock returns on firm characteristics with the Predictor Envelope Regression (PER) model has four equations.

$$r_{i,t} = a_t + b_t' x_{i,t} + e_{i,t} \tag{4}$$

$$b_t = G_t \eta_t \tag{5}$$

$$\eta_t = (G_t' X_t' X_t G_t)^{-1} G_t' X_t' R_t \tag{6}$$

$$G_t = [\Sigma_{X_t R_t}, \Sigma_{X_t}\Sigma_{X_t R_t}, ..., \Sigma_{X_t}^{K-1}\Sigma_{X_t R_t}]. \tag{7}$$

Additionally, assume X_t is cross-sectionally standardized, G_t is $J \times K$, $K \leq min(J, N)$ and vector η_t is $K \times 1$. $\Sigma_{X_t R_t} = \frac{1}{N} X_t'(R_t - 1_N a_t)$ is a vector of cross-sectional covariances between stock returns and firm characteristics. $\Sigma_{X_t} = \frac{1}{N} X_t' X_t$ is the cross-sectional covariance matrix of the firm characteristics.

The PER specification begins with the same equation as the standard OLS regression specification. Equation (4) states cross-sectional variation in stock returns is proportional to cross-sectional variation in standardized firm characteristics.

Equation (5) introduces the additional structure on b_t that distinguishes the PER specification from OLS. It assumes that b_t belongs to a K dimensional subspace of X_t spanned by matrix G_t. Intuition for this assumption is explained below. Constraining b_t to belong to a subspace of X_t with dimension $K \leq min(N, P)$ is the PER model's device for estimating the standard linear regression specification when $N < P$. Since b_t requires K spanning vectors, we can map the original $P > N$ variables composing X_t into $K \leq N$ new variables $X_t G_t$ and use these K new variables to estimate b_t instead of X_t. The K new variables $X_t G_t$ contain all of the information in X_t about the regression's slopes. And, importantly, the new variables $X_t G_t$ have an invertible covariance matrix because $K \leq N$. An OLS regression of stock returns on $X_t G_t$ yields regression coefficients η_t, which we can left multiply η_t by G_t to recover the b_t slopes for the infeasible regression of stock returns on X_t. Equation (6) states that η_t is the slope from the OLS regression $r_{i,t} = a_t + \eta_t' Z_t + e_{i,t}$ where $Z_{i,t} = G_t' X_t$.

Equation (7) describes the PER model's assumptions regarding the subspace of X_t spanning b_t by specifying G_t. Specifically, equation (7) states the subspace of X_t spanning b_t is the K order Krylov matrix specified by the characteristics' covariance matrix Σ_{X_t} and the vector of return and characteristic covariances $\Sigma_{X_t R_t}$. The sample Krylov matrix $G_t = [\Sigma_{X_t R_t}, \Sigma_{X_t}\Sigma_{X_t R_t}, ..., \Sigma_{X_t}^{K-1}\Sigma_{X_t R_t}]$ consistently estimates the subspace of X_t spanning b_t (Cook et al., 2007; Cook, 2018). Note that G_t is not unique because $G_t O$, with O orthogonal, produces the same solution for b_t.

Höskuldsson (1988) provides three intuitive explanations for the specification of G_t. First, G_t forms the K-variable linear combination of X_t with the smallest sum-of-squares prediction of R_t. Second, G_t generates the K combinations of X_t that have maximal covariance with R_t. And, third, columns of G_t extract the K largest, common factors in the univariate covariances between R_t and X_t.

The month t cross-sectional PER regression of stock returns on firm characteristics can be implemented in three steps after fixing a value for K. The parameter K can be estimated with the procedure in appendix A or given a user-chosen fixed integer value representing the number of repackaged variables the PER model uses to estimate b_t. The implementation's three steps follow.

1. Compute the columns of $G_t = [g_{1,t}, g_{2,t}, ..., g_{K,t}]$ sequentially. Let $g_{1,t} = \Sigma_{X_t R_t}$, then let $g_{k,t} = \Sigma_{X_t} g_{k-1,t}$ for $k = 2, ..., K$.
2. Run a cross-sectional regression of R_t on $X_t G_t$, i.e. $R_t = a_t + (X_t G_t)\eta_t + e_{i,t}$. Keep the slope vector η_t.
3. Compute $b_t = G_t \eta_t$.

The steps of the procedure summarize how the PER model estimates linear regression slopes. First, the PER model compresses information about cross-sectional variation in stock returns scattered across the P original firm characteristics into a sufficiently small number of variables for an OLS regression of stock returns on the compressed variables to be feasible. The "compression" matrix is G_t and the compressed variables are $X_t G_t$. Second, the PER model regresses stock returns on the compressed variables. The slope for the regression of stock returns on the compressed variables is η_t. Third, the PER model decompresses the slopes from the regression of stock returns on compressed characteristics into slopes for the original firm characteristics where the decompression of η_t is $b_t = G_t\eta_t$.

The compression metaphor also provides intuition for how varying K influences the slopes PER produces for b_t. Small values for K yield more compressed estimates of the original characteristics' information about the cross-section of stock returns. Suppose the value chosen for K is less than the population value of K. In this case, the PER model's compression of X_t discards some information about the covariance between stock returns and X_t, and the PER model's estimate of b_t will omit the discarded information. Next, if the value chosen for K equals the population value of K, then the PER model compresses X_t without discarding relevant information about b_t or keeping information irrelevant for estimating b_t. Last, suppose the value chosen for K exceeds the population value of K. In this scenario, the PER model compresses all information necessary for estimating b_t, but the PER model also compresses some variation in X_t that does not contain information about b_t.

The intuition from Höskuldsson (1988) about G_t also explains the order in which PER compresses information about characteristics' covariances with stock returns. For each value of K, PER compresses as much information as possible about characteristics' covariances with the cross-section of stock returns. The first column of G_t compresses X_t into the factor with maximal covariance with the cross-section of returns. The second column of G_t compresses X_t into the factor with the second-most covariance with stock returns. Column k of G_t compresses X_t into the factor with the k-th most covariance to the cross-section. The columns of G_t construct factors that have decreasing covariances with the cross-section of stock returns (De Jong, 1993). Since G_t extracts factors from X_t in decreasing order of their covariance with the cross-section, G_t collects as much information about covariance between returns and characteristics as possible for each value of K.

2.3 Modified Fama–Macbeth Regression

The Fama–Macbeth procedure's difficulties with panels where the number of characteristics is large or exceeds the number of cross-sectional observations is because of the procedure's use OLS to estimate cross-sectional regression slopes. A natural solution to running Fama–Macbeth regressions with more characteristics is replacing the OLS regression model with another regression model better suited to estimation tasks with many covariates relative to the number of observations. This section describes a modified Fama–Macbeth regression using the PER model from Cook et al. (2013) instead of OLS to estimate cross-sectional quantities. The modified Fama–Macbeth regression is the same in all respects except for its use of PER to find cross-sectional slopes instead of OLS.

The modified Fama–Macbeth procedure estimates stocks' expected returns in two steps. First, run monthly cross-sectional PER regressions of stock returns on firm characteristics. The regression for month t is

$$r_{i,t} = a_t + \sum_{j=1}^{J} \hat{b}_{j,t}^{PER} x_{i,t,j} + e_{i,t}. \tag{8}$$

Second, compute the sample average of the monthly, cross-sectional PER regression slopes:

$$\hat{b}_j^{PER} = \frac{1}{T} \sum_{t=1}^{T} \hat{b}_{j,t}^{PER}. \tag{9}$$

where variable $b_{j,t}^{PER}$ is the month t cross-sectional PER regression slope for characteristic j. And, variable $\hat{b}_j^{PER}$ is the time series average of the cross-sectional slopes PER slopes $\hat{b}_{j,t}^{PER}$. Standard errors and other summary statistics reported for the standard Fama–Macbeth regression model can be computed in the same manner for the modified Fama–Macbeth regression model.

3 Empirical Analysis

This section studies the cross-sectional relationship between 3,655 characteristic interactions and expected stock returns with the paper's envelope modification of the Fama–Macbeth regression model.

3.1 Data

This paper uses a collection of typical firm characteristics from other recent publications examining the relationship between the cross-section of expected returns and many firm characteristics. The vast majority of the paper's characteristics are from Green et al. (2017), which is a standard sample of characteristics for high dimensional cross-sectional studies, e.g. Han et al. (2019). The remaining characteristics are from Freyberger et al. (2020), which is also a standard characteristic sample, e.g. Kozak et al. (2020) and Kelly et al. (2019). Characteristic definitions follow the descriptions available in Green et al. (2017) and Freyberger et al. (2020). When additional implementation details are necessary I refer to the articles cited by Green et al. (2017) and Freyberger et al. (2020).

Table 1 lists the characteristics I use to examine the cross-sectional relationship between stock returns and characteristic interactions. The sample includes a combination of well-studied and less-studied characteristics. Characteristics are from both published articles and unpublished working papers. Some characteristics are from articles published some time ago, and other characteristics are from more recent publications. Sufficiently precise characteristic definitions for implementation and replication purposes are available in the appendices of Green et al. (2017) and Freyberger et al. (2020). The earliest characteristics are from 1977. The most recent characteristics are from 2016. The characteristics represent all categories listed in the classification scheme from Harvey et al. (2016).

The paper uses the CRSP database for stock returns and the Compustat database for other financial information. The sample begins with all common stocks traded on the NYSE, AMEX, or NASDAQ listed in CRSP. I keep stocks with month-end market values in CRSP and a non-missing value for common equity in their annual financial statements. I merge the Compustat database on the remaining CRSP sample of monthly stock returns. Month t characteristics use information available at the end of month $t-1$. I assume annual financial statements are available six months after a firm's fiscal year-end. I assume quarterly financial statements are available four months after the end of a firm's fiscal quarter. These assumptions for the alignment of stock prices and financial statements are standard and follow Green et al. (2017). Characteristics are updated monthly. The paper's sample is from January 1980 to December 2017.

I use the procedure from Green et al. (2017) to fill missing firm characteristic observations. Each month, I first winsorize characteristics at the 1st and 99th percentiles of their monthly values. Next, I cross-sectionally standardized the characteristics to have zero mean and unit standard deviation. Last, I assign missing characteristic observations the value zero in the post-standardized data-set. This approach to filling missing data points is the zero-order regression method from Wilks (1932). Filling missing firm characteristics is necessary. Most firms have a few characteristics with missing observations. Cross-sectional regressions excluding firms with missing characteristics would contain a negligible fraction of domestic publicly traded equity.

I build the panel of characteristic interactions used for the paper's results with the post-standardization characteristics before filling missing values with zero. The paper's interactions include all unique, two characteristic interactions of the original 85 characteristics in table 1. I've chosen this procedure for generating interactions because this is the standard procedure for generating interactions among variables in economic and financial studies. I do not include each characteristic's interaction with itself. The resulting panel has 3,655 cross-characteristic interactions, which I cross-sectionally standardize and assign the value zero to missing interaction values.

The paper reports results for several samples of domestic stocks. The five stock samples used for the paper's results are all stocks, all stocks without microcaps, large stocks, midcap stocks, and small stocks. Microcap stocks are stocks with market capitalizations below the 20th quantile of NYSE-trade stocks. Small stocks have market capitalizations below the 30th quantile of NYSE-traded stocks. Midcap stocks have market capitalizations above the 30th quantile and below the 70th quantile of NYSE-trade stocks. Large stocks have market capitalizations above the 70th quantile of NYSE traded stocks. NYSE market capitalization quantiles are from Ken French's monthly size deciles break-point data file.

3.2 Expected Return Estimates

I evaluate the relationship between characteristic interactions and the cross-section of expected returns by comparing stocks' realized returns to out-of-sample estimates of stocks' expected returns built with the 3,655 characteristic interactions and the Fama-Macbeth Envelope model. I use the procedure from Lewellen (2015) to construct out-of-sample estimates of stocks' expected returns. I form stocks' expected return estimates for month t in three steps. First, I use the PER model to run cross-sectional regressions of stock returns on characteristic interactions for months $s \in t-120, ..., t-1$. The specification equation for the month s cross-sectional PER regression is

$$r_{i,s} = a_s + \sum_{j_1,j_2,j_1 \neq j_2}^{J} b_{s,j_1,j_2} x_{i,s,j_1,j_2} + e_{i,s} \tag{10}$$

where x_{i,s,j_1,j_2} is the cross-sectionally standardized interaction of characteristics j_1 and j_2 for firm i. Second, I estimate each month t slope $\hat{b}_{t,j_1,j_2}$ with

$$\hat{b}_{t,j_1,j_2} = \frac{1}{120} \sum_s b_{s,j_1,j_2} \tag{11}$$

where $\frac{1}{120} \sum_s b_{s,j_1,j_2}$ is the time-series average of past b_{s,j_1,j_2} slopes. Third, I create month t expected return estimates with

$$\hat{r}_{i,t} = \sum_{j_1,j_2,j_1 \neq j_2}^{J} \hat{b}_{t,j_1,j_2} x_{i,t,j_1,j_2}. \tag{12}$$

Conceptually, the procedure uses the average cross-sectional relationship between characteristic interactions and expected returns over the previous ten years to estimate stocks' expected returns for month t.

The PER model's envelope dimension, parameter K in section 2.2, is fixed at 12 for the entire sample period. I estimate K with a three-step procedure. First, for each month t from January 1980 to December 1989 I use a standard sequential F-test from Osten (1988) to estimate the optimal envelope dimension, $\hat{K}_t$, for a PER regression of month t stock returns on characteristic interactions. The definition of the F-test is available in appendix

A. Second, I compute the time-series average of the monthly envelope dimension estimates, $\bar{K}_t = \frac{1}{120}\sum_t \hat{K}_t$, from January 1980 to December 1989. Third, I round $\bar{K}_t$ to the nearest integer.

3.3 Portfolios

This section uses portfolios to examine the information contained in characteristic interactions about the cross-section of expected returns. I use the interaction-based expected return estimates from section 3.2 to gather information from the entire collection of characteristic interactions into a single variable suitable for building portfolios. At the end of each month, stocks are assigned to decile portfolios according to their interaction-based expected return estimates for the following month. The first decile portfolio holds stocks with the lowest expected returns, and the tenth decile portfolio holds stocks with the highest expected returns. Portfolios are constructed monthly from January 1990 to December 2018.

Table 2 reports results for decile portfolios built from sorts on the interaction-based estimates of expected returns. The table's portfolios include all stocks except for microcap stocks. The table's results show that interactions contain economically important information about the cross-section of expected returns among both smaller stocks and larger stocks. Panel A reports results for equal-weight portfolios. The average returns of the equal-weight portfolios in Panel A increase smoothly from portfolio one to portfolio ten. A standard long-short portfolio long portfolio ten and short portfolio one from the equal-weight sort has an average monthly return of 1.24%. The t-statistic for the long-short portfolio's average return is 5.46. The Sharpe ratio for the long-short portfolio is 1.03. Panel B reports results for value-weight portfolios. The average returns for the value-weight portfolios increase from portfolio one to portfolio ten. The value-weight long-short portfolio has an average monthly return of 0.91%. The value-weight long-short portfolio's t-statistic is 3.75. And the value-weight long-short portfolio's Sharpe ratio is 0.71.

Table 3 reports results for long-short portfolios constructed from portfolios one and ten of sorts within several market capitalization subsamples of stocks. The table also reports results for long-short portfolios built with portfolios one and ten from decile sorts on stocks' estimates of their expected returns computed with the original 85 characteristics.

The expected return estimates using the original characteristics are made with the same procedure as the interaction-based expected return estimates, except for the procedure's first step. The monthly cross-sectional regressions of stock returns on the original characteristics use OLS instead of PER.

The results in table 3 show characteristic interactions contain economically important information about expected returns for firms of all sizes. Panel A reports the returns of equal-weight long-short portfolios for sorts on both interaction-based estimates of expected returns and characteristic-based estimates of expected returns. The average returns of the equal-weight long-short portfolios for interactions and characteristics are relatively similar. The standard deviations for the interaction long-short portfolios are about 30% less than the standard deviations for the characteristic long-short portfolios. The Sharpe ratios for the interaction long-short portfolios are also about 40% greater than the Sharpe ratios for the characteristic long-short portfolios. The t-statistics for the equal-weight long-short interaction portfolios all exceed three. Panel B reports long-short portfolios for value-weight decile portfolios. The average returns of the value-weight long-short portfolios for the interactions and characteristics are all positive and similar in magnitude. The standard deviations for the interaction long-short portfolios are noticeably less than for the characteristic portfolios. The Sharpe ratios for the interaction long-short portfolios in Panel B are about 20% greater than the Sharpe ratios for the corresponding characteristic long-short portfolios. The t-statistics for all of the value-weight interaction portfolios exceed three except for the large stock long-short portfolio, which has a t-statistic of 2.70.

3.4 Multifactor regressions

Next, I examine how recent multifactor models account for the average excess returns associated with characteristic interactions' long-short portfolio. I report results for the all-but-microcap characteristic interaction portfolios. I consider four specifications: the Carhart (1997) four-factor model, Hou et al. (2015) q-factor model, Fama and French (2015) five-factor model, and Fama and French (2015) five-factor with the winners-minus-losers factor. I run multifactor model regressions for each of the factor models' standard specifications. I also run multifactor model regressions with the characteristic long-short portfolio as an additional factor. The former regressions directly measure how the multifactor models account for the average returns associated with characteristic interactions. The latter regression models also account for information present in the paper's panel of 85 characteristics, but absent from the multifactor models. Overall, the section's results indicate characteristic interactions are associated with information in the cross-section of expected returns not accounted for by the Carhart (1997) factor model, Hou et al. (2015) factor model, Hou et al. (2015) factor model, and the characteristic long-short portfolios.

Table 4 reports results for the multifactor regressions of the interaction long-short portfolios' returns on the Carhart (1997) and Hou et al. (2015) factor models. Panel A reports results for the Carhart (1997) four-factor model. Panel A shows that the characteristic interaction long-short equal-weight and value-weight portfolios are associated with large and significant average risk-adjusted returns for the Carhart (1997) models. The Carhart (1997) factor model regressions including the characteristic long-short portfolio do not substantially affect the risk-adjusted returns of the interaction long-short portfolios. The long-short interaction portfolio has effectively zero loadings on the market and size factors, a negative loading on the value factor, and a positive loading on the momentum factor.

The results are similar in table4 Panel B, where results for the Hou et al. (2015) four-factor model are reported. The interaction portfolios have significant, positive risk-adjusted returns for the Hou et al. (2015) factor model. Including the characteristic long-short portfolios as additional factors reduce the interaction portfolios' risk-adjusted returns, but the interaction portfolios' risk-adjusted returns are still large, positive, and significant. The equal-weight and value-weight long-short interaction portfolios do not load on the market equity factor in a consistent manner across the regressions. The long-short interaction portfolios' load negatively on the investment factor and load positively on the return-on-equity factor. The loadings on the investment and return-on-equity factors are not significant for three of the four included regressions.

Table 5 reports results for the multifactor regressions of the interaction long-short portfolios' returns on the Fama and French (2015) five-factor model and Fama and French (2015) five-factor model with the winners-minus-losers factor. Panel A shows that the interaction long-short portfolios have positive, significant risk-adjusted returns for the Fama and French (2015) five-factor model. The interaction portfolios have negative loadings on the market factor, negative and significant loadings on the value factor, negative loadings on the profitability factor, and positive loadings on the investment factor. Including the characteristic long-short portfolio in the Fama and French (2015) five-factor model regression reduces the average risk-adjusted returns of the interaction long-short portfolios. However, the interaction portfolios still accrue relatively large and significant risk-adjusted returns.

Table 5 Panel B reports results for the multifactor regression of the interaction long-short portfolios' returns on the Fama and French (2015) five-factor model plus the winners-minus-losers momentum factor. The interaction long-short portfolios have positive, significant returns after adding the momentum factor to the five-factor model. The most material change when including the momentum factor in the Fama and French (2015) specification are the changes in the value and investment factor loadings. After including the momentum factor, the value factor loadings are reduced towards zero and are no longer significantly different from zero. The investment factor loadings also decrease slightly, but their significance does not change. The long-short interaction portfolios' returns have positive and significant loadings on the momentum factor. Adding the characteristic long-short portfolios to the multifactor regressions results in a roughly 0.15 percentage point decrease in the interaction portfolios' risk-adjusted returns.

3.5 Cross-sectional Regressions

This section uses statistics from cross-sectional regressions of realized returns on the interaction-based estimates of expected returns to evaluate characteristic interactions' information about the cross-section of expected returns. Table 6 reports results for Fama-Macbeth regressions of stock returns on four different expected return estimates. The first two expected return estimates are the interaction-based estimates and characteristic-based estimates. The third expected return estimate averages each stock's interaction- and characteristic- based estimates. Each averaged expected return estimate is $\hat{r}_{i,t}^{average} = (1/2)\hat{r}_{i,t}^{interact} + (1/2)\hat{r}_{i,t}^{character}$. The fourth expected return estimate is a combination forecast using the interaction- and characteristic-based estimates. Each combination expected return estimate is $\hat{r}_{i,t}^{combo} = \bar{\theta}_{t-120,t-1}^{interact}\hat{r}_{i,t}^{interact} + \bar{\theta}_{t-120,t-1}^{character}\hat{r}_{i,t}^{character}$ where $\bar{\theta}_{t-120,t-1}^{interact}$ and $\bar{\theta}_{t-120,t-1}^{character}$ are time-series averages of slopes from cross-sectional regressions of stock returns on the both the interaction- and characteristic-based expected returns for the periods $t-120, ..., t-1$. All of the Fama–Macbeth regressions uses the 1990 to 2017 sample except the regressions of returns on the combination expected return estimates, which use the 2000-2017 sample.

Column one from table 6 reports the slopes from Fama–Macbeth regressions of stock returns on the interaction-based expected return estimates. Column one shows a positive and significant relationship between realized stock returns and the interaction-based expected return estimates across stocks of all sizes. Since column one slopes are from Fama-Macbeth regressions of realized returns on expected return estimates, the slopes also measure how the expected return estimates vary cross-sectionally relative to realized returns (Lewellen, 2015). Since each panel's slope is between zero and one the interaction-based expected return estimates vary somewhat more cross-sectionally than realized returns.

Column two from table 6 reports the slopes from Fama–Macbeth regressions of stock returns on the characteristic-based expected return estimates. The slopes for the characteristic-based expected returns are positive and significant for most of the panels. The slope is positive but not significant for the large stock sample. Since column two slopes are also from Fama-Macbeth regressions of realized returns on expected return estimates, the slopes measure how the expected return estimates vary cross-sectionally relative to realized returns. The slopes are all between zero and one, which means the characteristic-based expected return estimates vary more than realized returns.

Column three from table 6 reports the slopes from Fama–Macbeth regressions of stock returns on both the interaction- and characteristic-based expected return estimates. The regression slopes measure each expected return estimate's incremental information about realized returns. All of the column three slopes for the interaction-based expected return estimates are positive and significant. The column three slopes for the characteristic-based expected return estimates are also positive and significant except for the large stock sample slope in Panel B. The column three results show both characteristic interactions and the original characteristics contain incremental information about expected returns.

Columns four from table 6 reports the slopes from Fama–Macbeth regressions of stock returns on the averaged interaction and characteristic expected return estimates. The averaged expected return estimates tell us whether the interaction- and characteristic-based expected return estimates are collectively better than the individual interaction- and characteristic-based expected return estimates. The slopes in column four are all positive and significant. Additionally, the slopes are consistently closer to one than the univariate interaction and characteristic slopes reported in columns one and two. This result says the averaged expected return estimates' cross-sectional variance better tracks realized returns' cross-sectional variance than the interaction and characteristic expected return estimates' cross-sectional variance.

Column five from table 6 reports the slopes from Fama–Macbeth regressions of stock returns on combination forecasts of stock returns using both the interaction and characteristic expected return estimates. The combination forecasts also tell us how well the interaction- and characteristic-based expected return estimates complement one another. The combination forecast slopes are positive and significant for all samples except the large stock sample, which has a positive but not significant coefficient. Additionally, the combination forecast slopes are consistently closer to one than the univariate interaction and characteristic slopes reported in columns one and two for all panels except the large stock panel.

Overall, the results in table 6 show characteristic interactions contain information about stocks' expected returns and that interactions contained incremental information about expected returns over the information present in characteristics without interactions. Additionally, table 6 indicate information in characteristics and their interactions is complementary with both collections of variables providing different information about the cross-section of expected returns.

3.6 Fama–Macbeth Envelope Regression Slopes

This section estimates which characteristic interaction slopes contain incremental and significant information about the cross-section of expected returns after controlling for other characteristic interactions and the original firm characteristics. Overall, the section's results show roughly 100 interactions contribute incremental information to expected returns. The

Fama–Macbeth envelope regression's cross-sectional specification is

$$r_{i,t} = a_t + \sum_{j_1,j_2,j_1 \neq j_2}^{J} b_{t,j_1,j_2} x_{i,t,j_1,j_2} + \sum_{j} b_{t,j} x_{i,t,j} + e_{i,t} \tag{13}$$

where x_{i,t,j_1,j_2} is the standardized interaction of characteristics j_1 and j_2 and $x_{i,t,j}$ is standardized characteristic j. The specification includes both characteristic interactions and the original characteristics so that the interactions' slope estimates are computed after controlling for both other interactions and the original characteristics. The regression's sample period is January 1980 to December 2017.

Figure 1 reports the Fama–Macbeth envelope regression's point estimates for the 3,655 characteristic interactions' slopes within the sample of all stocks except microcaps. Figure one shows positive and negative slopes are spread evenly across the interactions. Larger slopes in absolute value concentrate somewhat more around the momentum, size, and ipo characteristics. The prominent size interaction slopes are consistent with previous research showing firm size influences the effect of many characteristics on the cross-section. Figure 2 reports t statistics for the interactions' slopes. Figure 2 shows the values for the slopes t-statistics are fairly evenly dispersed across the characteristics.

Table 7 reports statistics for the number of interactions with p-values below standard significance levels. Panel A reports the total number of p-values below the 0.001, 0.01, and 0.05 significance levels for several different stock samples. At the 0.001 significance level between 26 and 103 interactions are significant. At the 0.01 significance level roughly 100-200 interactions are significant. At the 0.05 level roughly 300-450 interactions are significant. Overall, Panel A shows many characteristic interactions have slopes that are significantly different from zero at standard significance levels.

Table 7 Panel B reports the total number of interactions with p-values below a given significance level minus the number of expected type-I errors at the same significance level. For the 0.001, 0.01, and 0.05 significance levels the number of expected type-I errors is 4, 37, and 183, respectively. Panel B shows many more interactions have slopes that are significantly different from zero at each of the significance than could be accounted for by type-I errors alone.

Table 7 Panel C reports the percent of interactions with p-values below a given significance level at the given significance levels. Panel C is another way of comparing the number of significant interactions to the number of expected type-I errors. A percent greater than a significance level shows more interactions have significant slopes at the given level than would be produced by type-I errors. The percent of significant interactions at each given level is much greater than the significance levels themselves.

Table 8 reports each characteristic's number of interactions with p-values less than 0.01. Each row reports interaction counts for one characteristic. Each column reports results for a particular sample of stocks. For instance, the "2" in the absacc row and all stock column means two interactions with the absacc characteristic have p-values below 0.01. The table's results show all characteristics are associated with at least one significant interaction and that significant interactions are generally dispersed across the 85 characteristics. A few characteristics like baspread, eps, ipo, and returnvol are associated with more significant interactions than other characteristics.

Table 9 presents p-values for a statistical test of whether or not a characteristic produces at least one interaction with a non-zero slope. The test provides a formal means of determining which characteristics produce interactions with non-zero slopes. For characteristic j_1 the test's null hypothesis is

$$H_0 : b_{j_1,j_2} = 0 \text{ for all } j_2 \tag{14}$$

and the test's alternative hypothesis is

$$H_1 : b_{j_1,j_2} \neq 0 \text{ for some } j_2. \tag{15}$$

The test statistic itself is a Wald test given by

$$W = \mathbf{b}'_{j_1} \Sigma^{-1}_{\mathbf{b}_{j_1}} \mathbf{b}_{j_1} \tag{16}$$

where $\mathbf{b}_{j_1}$ is a vector of the modified Fama-Macbeth regression time-series average slopes for interactions including characteristic j_1 and $\Sigma_{\mathbf{b}_{j_1}}$ is the time-series covariance matrix of the cross-sectional slopes in $\mathbf{b}_{j_1}$. The test-statistic generalizes the standard Fama–Macbeth regression test of whether or not one slope is significantly different from zero.

Table 9 shows many characteristics are associated with at least one interaction variable whose slope is significantly different from zero. Most characteristics have at least one significant interaction among the all stock, all-but-micro, and small stock samples. A substantial number of characteristics have at least one significant interaction for the large and midcap stock samples. Overall, the table's results say most characteristics generate interactions' with incremental information about the cross-section of expected returns.

3.7 Robustness

This section examines the effect of the Fama–Macbeth Envelope model's envelope dimension on the paper's results. The envelope's dimension controls the number of variables the model uses to summarize the covariance between a cross-section of stock returns and characteristics. Overall, the section shows the paper's results are robust to the envelope dimension's specification. Envelopes with dimensions from one to twenty produce similar outcomes for the paper's main results. Envelope dimension values around the main results' envelope dimension estimate of 12 are typically somewhat stronger than results for lower envelope dimension values. Envelopes with smaller dimensions still capture economically significant variation in stock returns.

Table 10 reports a summary of the returns for long-short portfolios built with interaction-based expected return estimates and envelope dimensions ranging from one to twenty. The long-short portfolios are equally weighted and built in the same manner as the long-short portfolios in section 3.3. All of the long-short portfolios for envelope dimensions near twelve have similar returns, t-statistics, standard deviations, and Sharpe ratios. All of the envelope dimension' long-short portfolios have positive and significant average returns.

Table 11 reports risk-adjusted returns for the long-short portfolios with varying envelope dimensions. Risk-adjusted returns are reported for the following factor models: Carhart (1997) four-factor model (C4), Hou et al. (2015) q-factor model (HXZ4), Fama and French (2015) five-factor model (FF5), and Fama and French (2015) five-factor model plus WML (FF5 + WML). All of the envelope dimensions' long-short portfolios exhibit positive and significant risk-adjusted returns of across the factor models.

Table 12 reports Fama–Macbeth regressions of stock returns on both interaction and characteristic estimates of expected returns for a range of envelope values. Each row reports results for an interaction-based estimate of expected returns with a given envelope dimension. The characteristic-based expected return estimates are constant. The table shows characteristic interactions contribute incremental information to expected return estimates for envelopes with dimensions ranging from one to twenty. The characteristic-based expected return slopes are also positive and significant, meaning the characteristics contain some information not present in the interactions. The table's results also show that interactions and characteristics provide complimentary information across many envelope specifications.

Table 13 reports Fama–Macbeth regressions of stock returns on the interaction-based out-of-sample expected return estimates for envelope dimensions ranging from one to twenty. The table shows characteristics' interactions are a robust source of information about the cross-section of stock returns. Estimates for all of the envelope specifications are positively and significantly related to realized returns. The table also shows the expected return estimates vary somewhat more than than realized returns. Larger envelopes' estimates include somewhat more excess variation than smaller envelopes' estimates.

4 Conclusion

I estimate the effect of 3,665 characteristic interactions on stock returns with a Fama–Macbeth regression modified to accommodate cross-sections with more variables than observations. The modified Fama–Macbeth regression adds a collection of constraints, called an envelope, to the Fama-Macbeth procedure's cross-sectional regression model. The resulting cross-sectional model estimates the same slopes as the standard least squares model, provided the model's variables proxy for stocks' loadings for some factor model with less factors than the model's number of observations.

I find characteristic interactions are an important source of information about expected returns. A standard long-short portfolio constructed with out-of-sample estimates of stocks' interaction-based expected returns has a Sharpe ratio of 3.90. Characteristic interactions and characteristics are complimentary sources of information about expected returns. About

100 characteristic interactions have significant, incremental information about expected returns. The paper's results are robust to the specification of the envelope's dimension. The robustness results also indicate the proposed procedure for estimating the modified Fama-Macbeth regression's envelope dimension performs well.

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5 Tables and Figures

Table 1: This table reports the firm characteristics included in the paper's empirical results. The table also reports abbreviations and references for the firm characteristics.

Abbreviation	Firm characteristic	Reference
absacc	Absolute accruals	Bandyopadhyay et al. (2010)
acc	Working capital accruals	Sloan (1996)
age	Years since first compustat coverage	Jiang et al. (2005)
agr	Asset growth	Cooper et al. (2008)
ame	Asset to market	Bhandari (1988)
ato	Asset turnover	Soliman (2008)
baspread	Bid ask spread	Amihud and Mendelson (1989)
beta	Market beta	Fama and MacBeth (1973)
betasq	Market beta squared	Fama and MacBeth (1973)
bm	Book to market	Barr Rosenberg and Lanstein (1984)
bmia	Book to market, industry adjusted	Asness et al. (2000)
cash	Cash to assets	Palazzo (2012)
cashdebt	Cash to debt	Ou and Penman (1989)
cashpr	Cash productivity	S. and Rao (2009)
cfp	Operating cash flow to price	Desai et al. (2004)
cfpia	Operating cash flow to price, industry adjusted	Asness et al. (2000)
chatoia	Change in operating cash flow to price, industry adjusted	Soliman (2008)
chcsho	Change in common stock shares outstanding	Pontiff and Woodgate (2008)
chempia	Change in employees, industry adjusted	Asness et al. (2000)
chibqsup	Change in earnings surprise	Thomas and Zhang (2011)
chinv	Change in inventory	Thomas and Zhang (2002)
chmom	Change in 6-month momentum	Gettlemen and Marks (2006)
chpmia	Change in profit margin, industry adjusted	Soliman (2008)
chsaleqsup	Change in sales surprise	Thomas and Zhang (2011)
chtx	Change in tax expense	Thomas and Zhang (2011)
cinvest	Corporate investment	Titman et al. (2004)
convind	Convertible debt indicator	Valta (2016)

Table 1: (continued)

Abbreviation	Firm characteristic	Reference
cto	Capital turnover	Haugen et al. (1996)
currat	Current ratio	Ou and Penman (1989)
debtp	Debt to price	Gorodnichenko and Weber (2016)
depr	Depreciation to plants, property, and equipment	Holthausen and Larcker (1992)
divi	Dividend initiation indicator	Michaely et al. (1995)
divo	Dividend omission indicator	Michaely et al. (1995)
dolvol	Dollar trading volume	Chordia et al. (2001)
dpchgam_pchsale	Change in percent gross margin change less percent sales change	Abarbanell and Bushee (1998a)
dpia	Change in plants, property, and equipment plus inventory over assets	Lyandres et al. (2008)
dso	Log change in shares outstanding	Pontiff and Woodgate (2008)
durind	Indicator for members of durable goods industries	Sharpe (1994)
dy	Dividend to price	Litzenberger and Ramaswamy (1982)
egr	Growth in shareholder equity	Richardson et al. (2005)
ep	Earnings to price	Basu (1977)
eps	Earnings per share	Basu (1977)
gma	Gross profitability	Novy-Marx (2013)
gnpcorr	Correlation between GNP percent change and sales percent change	Sharpe (1994)
grcapx	Growth in capital expenditures	Anderson and Garcia-Feijóo (2006)
grltnoa	Growth in long-term net operating assets	Fairfield et al. (2003)
herf	Industry sales concentration	Hou and Robinson (2006)
hire	Employee growth rate	Belo et al. (2014)
ill	Illiquidity	Amihud (2002)
indmom	Industry momentum	Moskowitz and Grinblatt (1999)
invest	Capital expenditures and inventory	Chen and Zhang (2010)
ipo	Initial public offering	Loughran and Ritter (1995)
lev	Leverage	Bhandari (1988)
lgr	Growth in long-term debt	Richardson et al. (2005)
maxret	Max daily return in previous month	Bali et al. (2011)
mom12m	12-month momentum	Jegadeesh (1990)
mom1m	1-month momentum	Jegadeesh and Titman (1993)
mom36m	36-month momentum	Jegadeesh and Titman (1993)
mom6m	6-month momentum	Jegadeesh and Titman (1993)

Table 1: (continued)

Abbreviation	Firm characteristic	Reference
mve	Market value of equity	Banz (1981)
mveia	Market value of equity, industry adjusted	Asness et al. (2000)
nincr	Number of earnings consecutive earnings increases over past 8 quarters	Barth et al. (1999)
noa	Net operating assets	Hirshleifer et al. (2004)
ol	Operating leverage	Novy-Marx (2010)
operprof	Operating profitability	Fama and French (2015)
pchcapxia	Percent change in capital expenditures, industry adjusted	Abarbanell and Bushee (1998b)
pchcurrat	Percent change in current ratio	Ou and Penman (1989)
pchdepr	Percent change in depreciation	Holthausen and Larcker (1992)
pcheq	Percent change in book equity	Haugen et al. (1996)
pchgm_pchsale	Percent change in gross profit margin less percent change in sales	Abarbanell and Bushee (1998a)
pchquick	Percent change in quick ratio	Ou and Penman (1989)
pchsale_pchinvt	Percent change in sales less percent change in inventory	Abarbanell and Bushee (1998a)
pchsale_pchrect	Percent change in sales less percent change in receivables	Abarbanell and Bushee (1998a)
pchsale_pchxsga	Percent change in sales less percent change in selling and general administration expense	Abarbanell and Bushee (1998a)
pchsaleinv	Percent change in sales to inventory	Ou and Penman (1989)
pctacc	Percent accruals	Hafzalla et al. (2011)
pm	Price to cost	Soliman (2008)
pmia	Price to cost, industry adjusted	Soliman (2008)
quick	Quick ratio	Ou and Penman (1989)
rd	Indicator equaling 1 when change in R&D expense over total assets exceeds 5%	Eberhart et al. (2004)
retvol	Return volatility	Ang et al. (2006)
roeq	Return on quarterly equity	Hou et al. (2015)
roic	Return on invested capital	Brown and Rowe (2007)
salecash	Sales to cash	Ou and Penman (1989)
saleinv	Sales to inventory	Ou and Penman (1989)
salerec	Sales to receivables	Ou and Penman (1989)
secured	Long-term debt to secured debt	Valta (2016)
securedind	Secured debt indicator	Valta (2016)
sgr	Sales growth	Lakonishok et al. (1994)
sin	Alcohol, tobacco, and gambling industry indicator	Hong and Kacperczyk (2009)
tang	Tangiblility ratio	Almeida and Campello (2007)

Table 1: (continued)

Abbreviation	Firm characteristic	Reference
turn	Share turnover	Chordia et al. (2001)

Table 2: The table reports summary statistics for the excess returns of stocks sorted into decile portfolios on out-of-sample expected returns estimated with the modified Fama–Macbeth envelope regression model and 3,655 characteristic interactions. The out-of-sample expected return estimates' construction is described in section 3.2. The modified Fama–Macbeth regression model is defined in section 2.3. The characteristic interactions' construction is described in section 3.1. At the end of each month, I sort stocks into deciles according to their interaction-based expected return estimates for the following month. Next, I form equal-weight and value-weight portfolios for each decile and hold the portfolios until the end of the month. Sorts include all stocks except microcap stocks. Microcap stocks have market capitalizations below the 20th quantile of NYSE-trade stocks. I compute portfolio returns for the period of January 1990 through December 2017.

	Mean	t-stat.	Stdev.	Sharpe
Panel A. Equal weight portfolios				
1	-0.02	-0.04	7.81	-0.01
2	0.53	1.78	5.48	0.34
3	0.61	2.29	4.89	0.43
4	0.70	2.76	4.64	0.52
5	0.76	3.02	4.59	0.57
6	0.86	3.37	4.66	0.64
7	0.92	3.49	4.83	0.66
8	0.98	3.54	5.06	0.67
9	1.13	3.61	5.73	0.68
10	1.22	3.14	7.13	0.59
$10-1$	1.24	5.46	4.16	1.03
Panel B. Value weighted portfolios				
1	0.00	-0.01	6.62	0.00
2	0.66	2.67	4.51	0.51
3	0.58	2.53	4.22	0.48
4	0.58	2.60	4.08	0.49
5	0.64	2.87	4.07	0.54
6	0.76	3.42	4.04	0.65
7	0.72	3.17	4.19	0.60
8	0.79	3.17	4.58	0.60
9	0.86	3.23	4.89	0.61
10	0.90	2.67	6.20	0.50
$10-1$	0.91	3.75	4.43	0.71

Table 3: The table reports summary statistics for the excess returns of long-short portfolios built from deciles portfolios formed from sorting stocks according to their interaction-based expected return estimates for the following month. The long-short portfolios' constructions are standard. A long-short portfolio is long the decile portfolio holding stocks with the highest expected return estimates and short the decile portfolio with the lowest expected return estimates. I compute portfolio returns for the period of January 1990 through December 2017. The all stock sample includes all stocks passing the basic data screens in section 3.1. The all but microcaps sample omits microcap stocks, which have market capitalizations below the 20th percentile of NYSE-traded stocks. Small stocks have market capitalizations below the 30th quantile of NYSE-traded stocks. Midcap stocks have market capitalizations above the 30th quantile and below the 70th quantile of NYSE-trade stocks. Large stocks have market capitalizations above the 70th quantile of NYSE traded stocks.

	Characteristic interactions				Original characteristics			
	Mean	t(Mean)	Sd	Sharpe	Mean	t(Mean)	Sd	Sharpe
Panel A. Equal weight long-short portfolios								
All stocks	3.86	20.66	3.42	3.90	3.65	12.82	5.22	2.42
All but microcaps	1.24	5.46	4.16	1.03	1.20	3.53	6.25	0.67
Large	0.75	3.17	4.33	0.60	0.63	2.04	5.65	0.39
Medium	0.87	3.63	4.37	0.69	1.08	3.02	6.58	0.57
Small	4.48	21.07	3.89	3.98	4.40	14.59	5.53	2.76
Panel B. Value weight long-short portfolios								
All stocks	1.10	3.84	5.25	0.73	1.19	3.51	6.25	0.66
All but microcaps	0.91	3.75	4.43	0.71	1.00	2.85	6.45	0.54
Large	0.60	2.70	4.09	0.51	0.65	2.06	5.77	0.39
Medium	0.87	3.59	4.46	0.68	1.07	2.94	6.69	0.56
Small	1.99	7.96	4.59	1.50	2.30	7.42	5.68	1.40

Table 4: The table reports multifactor regressions of long-short interaction portfolios' monthly returns on the Carhart (1997) four-factor model and the Hou et al. (2015) four-factor model. The table also reports multifactor regressions with the equal-weight and value-weight long-short portfolios for the characteristic-based expected return estimates as additional factors. The long-short portfolio constructions are standard. *, **, and *** indicate significance at the 5%, 1%, and 0.1% levels, respectively based on heteroscedasticity t-statistics. 0.00 indicates a value less than 0.005 in absolute value. The factors are as follows: MKT = market excess return, SMB = "small minus big" size factor, HML = "high minus low" value factor, WML = "winner minus loser" momentum factor, CHAREW = long-short portfolio from equal-weight, decile sort on characteristic-based estimates of expected returns, CHARVW = long-short portfolio from value-weight, decile sort on characteristic-based estimates of expected returns, ME = market equity factor, IA = investment factor, ROE = return on equity factor.

	Equal weight		Value weight	
Panel A. Carhart (1997) four-factor model				
α (%)	0.95***	0.74***	0.85***	0.75**
MKT	−0.02	0.00	−0.02	−0.02
SMB	0.09	0.02	−0.04	−0.14
HML	−0.01	−0.06	−0.300**	−0.288*
WML	0.62***	0.27***	0.34***	0.13
CHAREW		0.32***		
CHARVW				0.21**
R^2 (%)	52.86	60.05	20.12	24.11
Panel C. Hou et al. (2015) four factor model				
α (%)	1.01***	0.66***	1.00***	0.78**
MKT	−0.11	−0.01	−0.11	−0.06
ME	0.34	0.03	0.07	−0.19
IA	−0.06	−0.03	−0.36 *	−0.25
ROE	0.52***	0.02	0.17	−0.05
CHAREW		0.49***		0.32***
CHARVW				
R^2 (%)	15.34	56.57	3.79	21.31

Table 5: The table reports multifactor regressions of long-short interaction portfolios' monthly returns on the Fama and French (2015) five factor model and Fama and French (2015) five factor model plus the WML factor. The table also reports multifactor regressions including the equal-weight and value-weight long-short portfolios for the characteristic-based expected return estimates as additional factors. The long-short interaction portfolio is long the tenth decile and short the first decile of all stocks but microcaps sorted according to their interaction-based expected return estimates. *, **, and *** indicate significance at the 5%, 1%, and 0.1% levels, respectively based on heteroscedasticity t-statistics. 0.00 indicates a value less than 0.005 in absolute value. The factors are as follows: MKT = market excess return, SMB = "small minus big" size factor, HML = "high minus low" value factor, RMW = "robust minus weak" profitability factor, "CMA" = "conservative minus aggressive" investment factor, WML = "winner minus loser" momentum factor, CHAREW = long-short portfolio from equal-weight, decile sort on characteristic-based estimates of expected returns, CHARVW = long-short portfolio from value-weight, decile sort on characteristic-based estimates of expected returns.

	Equal weight		Value weight	
Panel A. Fama and French (2015) five factor model				
α (%)	1.32***	0.73***	1.12***	0.81***
MKT	−0.16	−0.01	−0.13	−0.06
SMB	0.11	−0.07	−0.04	−0.18
HML	−0.50 ***	−0.21 **	−0.46 ***	−0.28
RMW	0.00	−0.16	−0.06	−0.05
CMA	0.60**	0.28*	0.12	−0.04
CHAREW		0.49***		
CHARVW				0.28***
R^2 (%)	12.59	58.85	8.69	23.25
Panel B. Fama and French (2015) five factor model with WML factor				
α (%)	0.97***	0.78***	0.92***	0.81***
MKT	−0.01	0.00	−0.05	−0.05
SMB	0.01	−0.05	−0.09	−0.17
HML	−0.12	−0.14	−0.25	−0.24
RMW	−0.17	−0.18	−0.15	−0.09
CMA	0.31**	0.26*	−0.04	−0.07
WML	0.61***	0.27***	0.34***	0.15
CHAREW		0.32***		
CHARVW				0.20**
R^2 (%)	54.93	61.80	20.61	24.27

Table 6: Fama-Macbeth regressions of stock returns on interaction-based expected return estimates, characteristic-based expected return estimates, averaged interaction and characteristic expected return estimates, and combination expected return estimates. $\hat{r}_{i,t}^{interact}$ is the interaction-based expected return estimates. $\hat{r}_{i,t}^{character}$ is the characteristic-based expected return estimates. $\hat{r}_{i,t}^{average} = (1/2)\hat{r}_{i,t}^{interact} + (1/2)\hat{r}_{i,t}^{character}$ is the averaged expected return estimate. $\hat{r}_{i,t}^{combo} = \bar{\theta}_{t-120,t-1}^{interact}\hat{r}_{i,t}^{interact} + \bar{\theta}_{t-120,t-1}^{character}\hat{r}_{i,t}^{character}$ where $\bar{\theta}_{t-120,t-1}^{interact}$ and $\bar{\theta}_{t-120,t-1}^{character}$ are time series averages of the slopes from cross-sectional regressions of stock returns on the two expected return estimates for the periods $t-120, ..., t-1$. *, **, and *** indicate significance at the 5%, 1%, and 0.1% levels. Regressions use the all-but-microcap sample of stocks.

	(1)	(2)	(3)	(4)	(5)
Panel A. All but microcaps					
$\hat{r}_{i,t}^{interact}$	0.31***		0.24***		
$\hat{r}_{i,t}^{character}$		0.38***	0.27**		
$\hat{r}_{i,t}^{average}$				0.51***	
$\hat{r}_{i,t}^{combo}$					0.81***
Avg. R^2	0.71	1.53	1.90	1.34	1.36
Panel B. Large					
$\hat{r}_{i,t}^{interact}$	0.23**		0.19*		
$\hat{r}_{i,t}^{character}$		0.23	0.20		
$\hat{r}_{i,t}^{average}$				0.36**	
$\hat{r}_{i,t}^{combo}$					0.25
Avg. R^2	1.08	1.91	2.66	1.79	1.65
Panel C. Medium					
$\hat{r}_{i,t}^{interact}$	0.27***		0.22***		
$\hat{r}_{i,t}^{character}$		0.34***	0.27**		
$\hat{r}_{i,t}^{average}$				0.48***	
$\hat{r}_{i,t}^{combo}$					0.97*
Avg. R^2	0.80	1.68	2.17	1.54	1.47
Panel D. Small					
$\hat{r}_{i,t}^{interact}$	0.58***		0.45***		
$\hat{r}_{i,t}^{character}$		0.71***	0.46***		
$\hat{r}_{i,t}^{average}$				0.90***	
$\hat{r}_{i,t}^{combo}$					0.84***
Avg. R^2	0.71	0.80	1.25	1.02	1.09

Figure 1: Characteristic interaction slopes from a Fama–Macbeth envelope regression of stock returns on 3,655 characteristic interactions and 85 original characteristics. The modified Fama–Macbeth envelope regression model is defined in 2.3. Each square represents the slope for an interaction generated by two characteristics. The color gradient representing interactions' slope values ranges from red for slopes above 0.03 to blue for slopes below -0.03. Lighter shades of red and blue represent intermediate slope values. Slopes equal to zero are represented by white. The sample period is January 1980 to December 2017. The Fama-Macbeth envelope regression includes all stocks except microcap stocks.

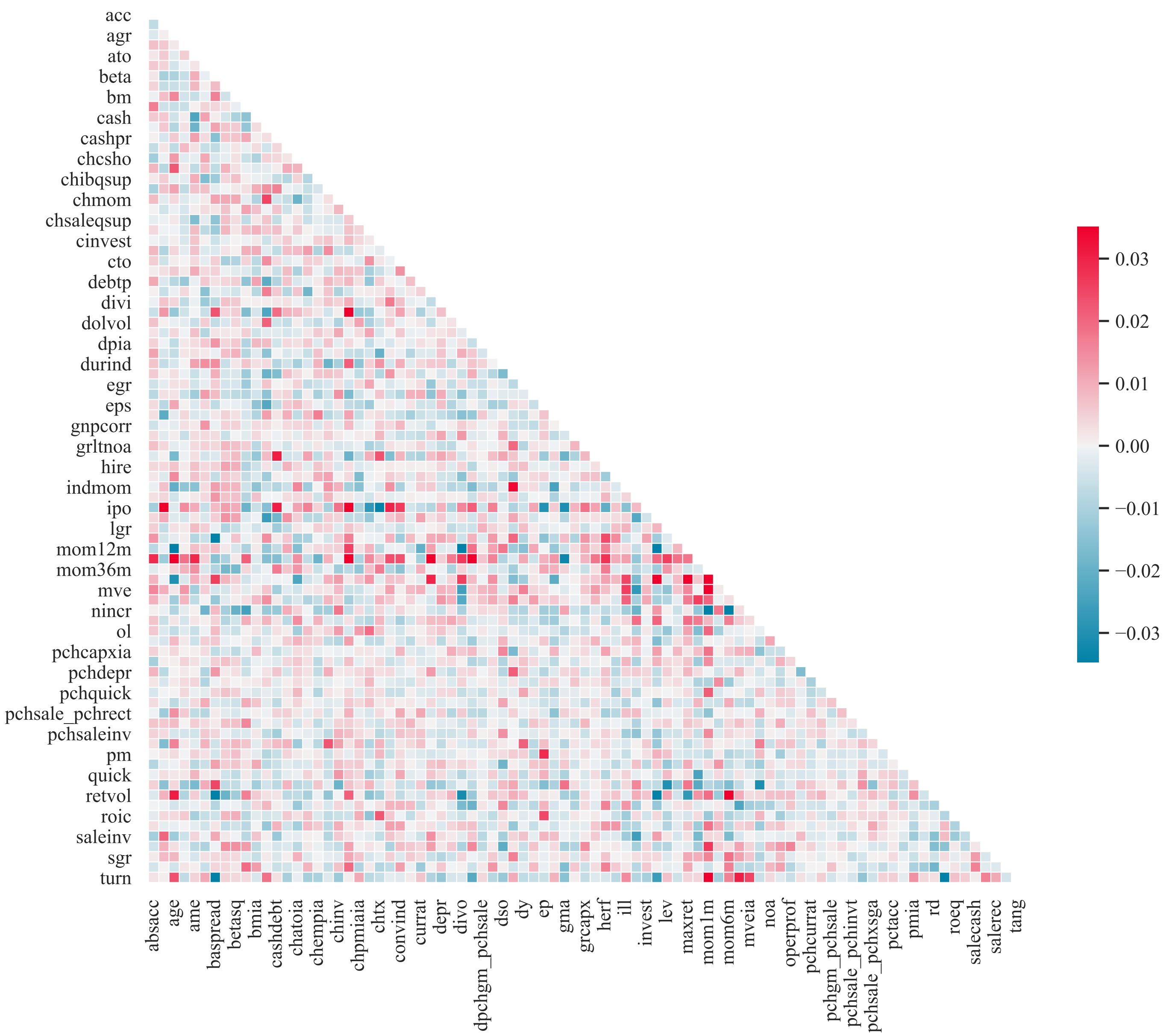

Figure 2: t-statistics for characteristic interaction slopes from a Fama–Macbeth envelope regression of stock returns 85 characteristics and the characteristics' 3,655 interactions. The modified Fama–Macbeth envelope regression model is defined in 2.3. The interactions and characteristics are defined in section 3.1. Each square represents the t-statistic for the slope of an interaction generated by two characteristics. The color gradient representing interactions' average slope values ranges from red for slopes above 0.03 to blue for slopes below -0.03. Lighter shades of red and blue represent intermediate slope values. The sample period is January 1980 to December 2017. The Fama-Macbeth envelope regression includes all stocks except microcap stocks.

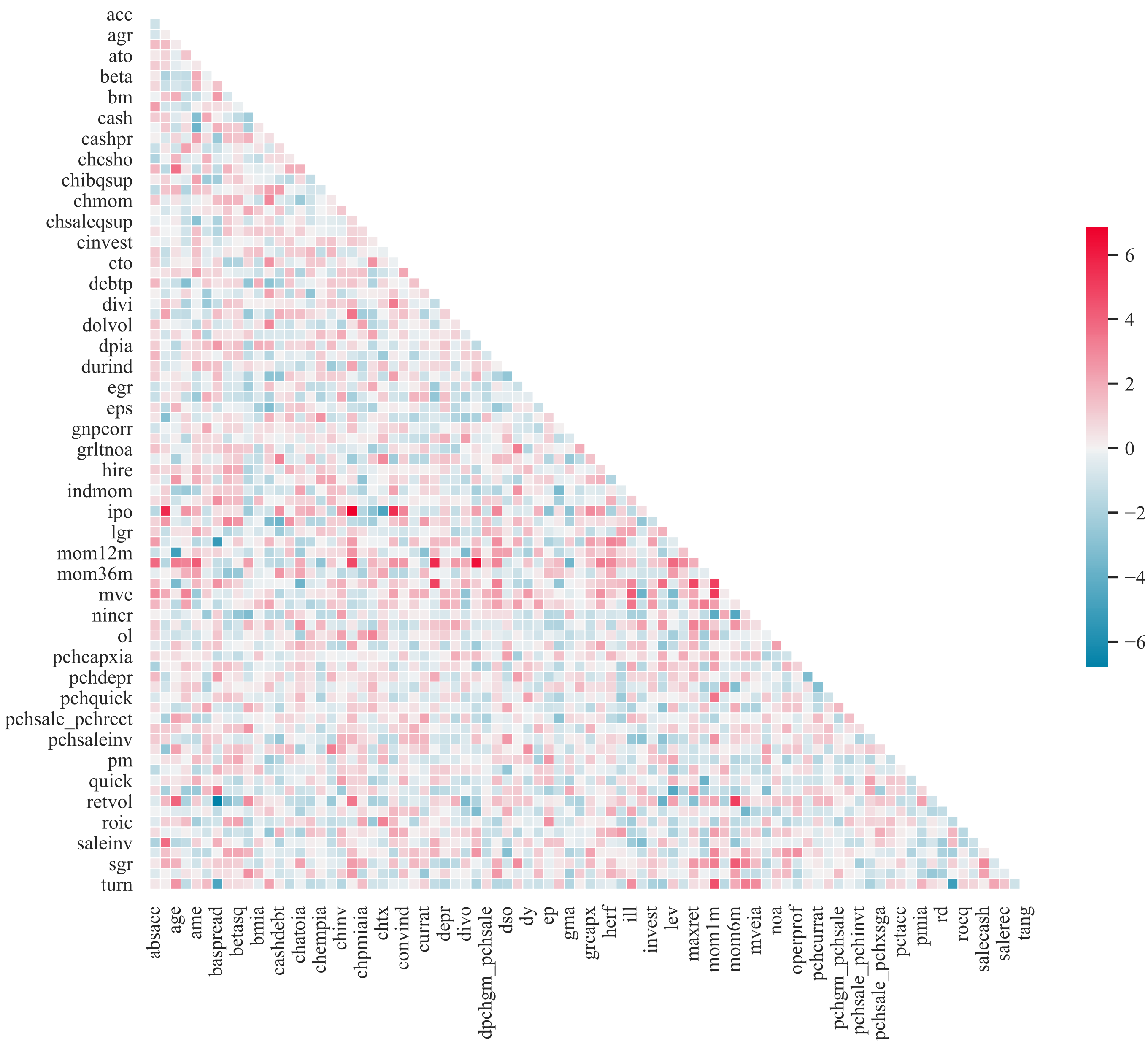

Table 7: Total p-values below a given significance level for interaction slopes from a modified Fama–Macbeth regression of stock returns on 3,655 interactions and 85 original firm characteristics. The modified Fama–Macbeth envelope regression model is defined in 2.3. The interactions and characteristics are defined in section 3.1. The interaction slopes are time-series averages of the Fama–Macbeth envelope regression model's cross-sectional regressions of stock returns on characteristics and characteristic interactions. Each row reports the number of p-values below a standard significance level. Each column reports results for a specific stock sample.

	All stocks	All-but-microcaps	Large	Medium	Small
Panel A. Total p-values below significance level					
p-value < 0.001	103	51	26	34	93
p-value < 0.01	207	154	88	99	196
p-value < 0.05	442	384	282	294	441
Panel B. Total p-values below significance level less expected type-I errors					
p-value < 0.001	99	47	22	30	89
p-value < 0.01	170	117	51	62	159
p-value < 0.05	259	201	99	111	258
Panel C. Percent p-values below significance level					
p-value < 0.001	2.82	1.4	0.71	0.93	2.54
p-value < 0.01	5.66	4.21	2.41	2.71	5.36
p-value < 0.05	12.09	10.51	7.72	8.04	12.07

Table 8: Each characteristic's number of interaction slopes with p-values < 0.01. Interaction slopes and p-values are from Fama–Macbeth envelope regressions of stock returns on 85 characteristics and the characteristics' 3,655 interactions. The Fama–Macbeth envelope regression model is defined in 2.3. The interactions and characteristics are defined in section 3.1. Each row reports the number of slopes with p-values below 0.01 for a given characteristic's interactions. Each column reports results for a specific stock sample.

	All stock	All-but-micro	Large	Medium	Small
absacc	2	2	2	0	2
acc	3	4	2	3	7
age	8	6	2	3	3
agr	5	4	5	4	5
ame	5	4	2	2	6
ato	0	0	2	1	0
baspread	12	6	2	5	11
beta	5	3	3	0	2
betasq	3	1	1	1	4
bm	11	5	2	4	6
bmia	0	0	0	2	2
cash	11	9	8	1	11
cashdebt	3	5	2	2	3
cashpr	3	1	0	0	1
chatoia	3	2	0	0	4
chcsho	3	1	3	0	4
chempia	0	3	0	1	0
chibqsup	3	1	1	1	3
chinv	1	1	0	5	1
chmom	9	6	9	8	7
chpmiaia	3	1	2	0	1
chsaleqsup	4	4	1	0	4
chtx	3	3	1	0	3
cinvest	2	3	4	2	2
convind	2	1	1	1	1
cto	1	2	1	0	2
currat	4	1	1	2	4
debtp	4	6	5	3	3
depr	0	1	0	1	0
divi	0	1	2	3	1
divo	7	5	1	3	8
dolvol	7	5	2	6	11
dpchgm__pchsale	0	0	1	0	2
dpia	2	4	3	2	2
dso	0	3	1	1	1
durind	4	3	1	2	3
dy	1	4	2	1	1
egr	2	0	1	1	2
ep	6	3	2	2	8
eps	10	4	1	0	11
gma	5	4	1	2	4
gnpcorr	2	0	3	0	2
grcapx	1	1	0	2	0
grltnoa	1	3	1	0	2
herf	0	4	2	3	2
hire	1	1	0	2	1
ill	7	5	1	1	7
indmom	8	6	3	6	8
invest	1	1	1	0	2
ipo	27	18	6	8	26
lev	10	8	2	5	9

Table 8: (continued)

	All stock	All-but-micro	Large	Medium	Small
lgr	3	1	3	1	5
maxret	7	7	2	4	6
mom12m	13	7	2	3	11
mom1m	20	24	20	17	18
mom36m	3	2	0	0	4
mom6m	14	16	11	11	12
mve	18	11	2	2	16
mveia	14	3	0	1	11
nincr	9	5	1	5	6
noa	5	3	3	3	6
ol	3	1	0	2	2
operprof	4	2	1	2	4
pchcapxia	0	1	0	0	1
pchcurrat	3	1	1	0	2
pchdepr	0	1	0	0	0
pchgm__pchsale	2	3	1	1	3
pchquick	1	1	0	0	1
pchsale__pchinvt	1	1	2	1	1
pchsale__pchrect	1	1	2	0	2
pchsale__pchxsga	4	1	1	0	4
pchsaleinv	1	3	0	1	1
pctacc	3	3	3	0	0
pm	2	0	1	3	3
pmia	3	1	0	0	2
quick	4	1	1	3	3
rd	9	3	1	4	10
retvol	13	10	4	12	10
roeq	9	2	1	1	7
roic	6	2	0	3	4
salecash	2	2	4	3	2
saleinv	5	3	1	1	5
salerec	2	4	1	3	3
sgr	4	5	2	4	2
tang	3	1	2	1	4

Table 9: p-values for tests of whether or not a given characteristic generates at least one interaction with a slope that is significantly different from zero. The interaction slopes are from a Fama–Macbeth envelope regressions of monthly stock returns on 85 characteristics and their 3,655 interactions. The modified Fama–Macbeth envelope regression model is defined in 2.3. The interactions and characteristics are defined in section 3.1. Each row reports p-values for tests of a given characteristic. For characteristic i the test's null hypothesis is $H_0 : b_{i,j} = 0$ for all $j = 1, ..., J$ where $b_{i,j}$ is the Fama–Macbeth regression slope for the interaction of characteristics i and j. The test's alternative hypothesis is $H_1 : b_{i,j} \neq 0$. The test is implemented with a Wald statistic defined in section 3.6. The test statistic is a generalization of the standard Fama-Macbeth t-test to hypotheses involving several slopes.

	All stock	All-but-micro	Large	Medium	Small
absacc	0.000	0.004	0.000	0.123	0.000
acc	0.000	0.000	0.000	0.001	0.000
age	0.000	0.000	0.002	0.000	0.000
agr	0.000	0.000	0.002	0.008	0.000
ame	0.000	0.000	0.002	0.003	0.000
ato	0.024	0.013	0.008	0.149	0.038
baspread	0.000	0.000	0.020	0.000	0.000
beta	0.000	0.000	0.133	0.063	0.000
betasq	0.000	0.001	0.301	0.086	0.000
bm	0.000	0.000	0.003	0.000	0.000
bmia	0.003	0.560	0.048	0.041	0.000
cash	0.000	0.000	0.000	0.115	0.000
cashdebt	0.000	0.000	0.011	0.181	0.000
cashpr	0.000	0.077	0.009	0.419	0.000
chatoia	0.001	0.001	0.125	0.091	0.001
chcsho	0.001	0.257	0.001	0.772	0.000
chempia	0.312	0.000	0.155	0.009	0.454
chibqsup	0.000	0.002	0.765	0.554	0.000
chinv	0.004	0.000	0.181	0.000	0.000
chmom	0.000	0.000	0.000	0.000	0.000
chpmiaia	0.063	0.020	0.094	0.195	0.221
chsaleqsup	0.000	0.013	0.004	0.006	0.000
chtx	0.000	0.002	0.241	0.160	0.000
cinvest	0.000	0.000	0.000	0.001	0.000
convind	0.010	0.089	0.080	0.013	0.016
cto	0.000	0.022	0.195	0.001	0.003
currat	0.000	0.001	0.096	0.039	0.000
debtp	0.000	0.000	0.000	0.000	0.000
depr	0.039	0.059	0.067	0.227	0.020
divi	0.011	0.007	0.048	0.001	0.002
divo	0.000	0.000	0.000	0.000	0.000
dolvol	0.000	0.000	0.000	0.009	0.000
dpchgm__pchsale	0.145	0.866	0.063	0.333	0.334
dpia	0.000	0.000	0.000	0.216	0.000
dso	0.099	0.000	0.006	0.132	0.027
durind	0.000	0.000	0.464	0.001	0.000
dy	0.000	0.000	0.000	0.056	0.003
egr	0.005	0.003	0.015	0.096	0.000
ep	0.000	0.000	0.039	0.017	0.000
eps	0.000	0.000	0.002	0.420	0.000
gma	0.000	0.000	0.000	0.001	0.000
gnpcorr	0.001	0.280	0.086	0.021	0.007
grcapx	0.065	0.231	0.415	0.000	0.092
grltnoa	0.000	0.001	0.163	0.026	0.000
herf	0.008	0.000	0.008	0.003	0.000
hire	0.244	0.025	0.190	0.016	0.214
ill	0.000	0.000	0.041	0.019	0.000
indmom	0.000	0.000	0.105	0.002	0.000

Table 9: (continued)

	All stock	All-but-micro	Large	Medium	Small
invest	0.000	0.174	0.000	0.146	0.000
ipo	0.000	0.000	0.000	0.000	0.000
lev	0.000	0.000	0.000	0.000	0.000
lgr	0.000	0.074	0.006	0.046	0.000
maxret	0.000	0.000	0.001	0.000	0.000
mom12m	0.000	0.000	0.000	0.000	0.000
mom1m	0.000	0.000	0.000	0.000	0.000
mom36m	0.000	0.021	0.007	0.391	0.000
mom6m	0.000	0.000	0.000	0.000	0.000
mve	0.000	0.000	0.011	0.001	0.000
mveia	0.000	0.000	0.001	0.114	0.000
nincr	0.000	0.000	0.254	0.000	0.000
noa	0.000	0.000	0.003	0.002	0.000
ol	0.001	0.104	0.335	0.000	0.000
operprof	0.000	0.066	0.013	0.007	0.000
pchcapxia	0.010	0.633	0.894	0.109	0.016
pchcurrat	0.005	0.017	0.005	0.029	0.012
pchdepr	0.092	0.224	0.026	0.118	0.262
pchgm_pchsale	0.000	0.043	0.234	0.463	0.000
pchquick	0.246	0.095	0.001	0.003	0.131
pchsale_pchinvt	0.009	0.032	0.034	0.046	0.000
pchsale_pchrect	0.089	0.049	0.026	0.031	0.002
pchsale_pchxsga	0.000	0.397	0.194	0.589	0.000
pchsaleinv	0.000	0.000	0.076	0.023	0.000
pctacc	0.001	0.125	0.009	0.489	0.004
pm	0.000	0.000	0.041	0.000	0.000
pmia	0.002	0.255	0.222	0.204	0.000
quick	0.000	0.004	0.319	0.005	0.000
rd	0.000	0.000	0.003	0.000	0.000
retvol	0.000	0.000	0.000	0.000	0.000
roeq	0.000	0.001	0.137	0.005	0.000
roic	0.000	0.001	0.103	0.000	0.000
salecash	0.000	0.000	0.000	0.000	0.000
saleinv	0.000	0.040	0.042	0.008	0.000
salerec	0.000	0.000	0.073	0.000	0.000
sgr	0.000	0.000	0.016	0.040	0.000
tang	0.001	0.000	0.013	0.010	0.000

Table 10: Summary statistics for the excess returns of long-short portfolios made with decile sorts on interaction-based expected return estimates and envelope dimensions ranging from one to twenty. The interaction-based expected return estimates are made with the procedure in section 3.2. The estimation procedure uses the Fama-Macbeth envelope regression model from section 2.3 to estimate the average cross-sectional relationship between 3,655 interactions and stock returns. The Fama-Macbeth envelope regression's envelope dimension is a tuning parameter, which controls the number of repackaged variables the model uses to represent cross-sectional information in the original variables about expected returns. Returns are reported in monthly percentage points. The long-short portfolio's associated decile portfolios are equally-weighted. The sample includes all stocks except microcap stocks.

Envelope	Mean	t(Mean)	Sd.	Sharpe
1	1.08	3.75	5.26	0.71
2	1.14	3.94	5.32	0.75
3	1.17	3.97	5.39	0.75
4	1.17	4.11	5.24	0.78
5	1.24	4.51	5.03	0.85
6	1.23	4.50	5.00	0.85
7	1.22	4.71	4.74	0.89
8	1.23	4.88	4.63	0.92
9	1.22	4.91	4.56	0.93
10	1.18	4.86	4.47	0.92
11	1.23	5.24	4.30	0.99
12	1.24	5.46	4.16	1.03
13	1.25	5.66	4.06	1.07
14	1.22	5.54	4.05	1.05
15	1.19	5.45	4.00	1.03
16	1.16	5.41	3.94	1.02
17	1.10	5.30	3.79	1.00
18	1.07	5.24	3.74	0.99
19	1.07	5.29	3.70	1.00
20	1.07	5.33	3.68	1.01

Table 11: Average risk-adjusted returns for long-short portfolios made with decile sorts on interaction-based expected return estimates and envelope dimensions ranging from one to twenty. The interaction-based expected return estimates are made with the procedure in section 3.2. The estimation procedure uses the Fama-Macbeth envelope regression model from section 2.3 to estimate the average cross-sectional relationship between 3,655 interactions and stock returns. The Fama-Macbeth envelope regression's envelope dimension is a tuning parameter, which controls the number of repackaged variables the model uses to represent cross-sectional information in the original variables about expected returns. Average risk-adjusted returns are reported for regressions of the long-short portfolios' excess returns on standard multifactor models for the cross-section of expected returns. The included factor models are the Carhart (1997) four-factor model (C4), the Hou et al. (2015) q-factor model (HXZ4), the Fama and French (2015) five-factor model (FF5), and the Fama and French (2015) five-factor model plus WML (FF5 + WML). The average risk-adjusted returns' $t-$statistics use heteroscedasticity robust standard errors. The long-short portfolios are made with equally weighted decile portfolios and all stocks except microcap stocks.

	C4		HXZ4		FF5		FF5 + WML	
Envelope	α	t(α)	α	t(α)	α	t(α)	α	t(α)
1	0.83	5.31	0.90	3.82	1.15	4.87	0.87	5.16
2	0.82	5.25	0.88	3.70	1.15	4.85	0.86	5.15
3	0.82	5.21	0.88	3.69	1.15	4.77	0.86	5.06
4	0.85	5.34	0.92	3.77	1.19	4.92	0.89	5.27
5	0.89	5.55	0.98	3.87	1.26	5.04	0.95	5.48
6	0.91	5.73	0.98	3.93	1.28	5.06	0.95	5.60
7	0.93	5.90	1.01	3.98	1.32	5.23	0.99	5.90
8	0.95	6.13	1.03	4.18	1.32	5.31	0.97	5.97
9	0.93	5.99	0.99	3.87	1.30	5.04	0.94	5.77
10	0.90	5.65	0.96	3.65	1.28	4.73	0.90	5.34
11	0.84	5.15	0.90	3.26	1.23	4.31	0.83	4.74
12	0.86	5.17	0.92	3.28	1.25	4.28	0.85	4.67
13	0.87	5.16	0.92	3.27	1.25	4.22	0.84	4.58
14	0.83	4.99	0.87	3.04	1.23	4.08	0.80	4.41
15	0.82	4.70	0.89	2.92	1.24	3.90	0.78	4.12
16	0.82	4.67	0.92	3.00	1.24	3.91	0.78	4.10
17	0.72	3.76	0.83	2.60	1.16	3.45	0.68	3.26
18	0.70	3.53	0.80	2.47	1.14	3.35	0.64	3.05
19	0.68	3.33	0.78	2.40	1.07	3.17	0.61	2.79
20	0.64	3.05	0.87	2.69	1.07	3.46	0.67	3.13

Table 12: Slopes from Fama-Macbeth regressions of stock returns on interaction- and characteristic-based estimates of expected returns where the interaction-based expected return estimates are computed with envelope dimensions from one to twenty. The interaction-based expected return estimates are made with the procedure in section 3.2. The estimation procedure uses the Fama-Macbeth envelope regression model from section 2.3 to estimate the average cross-sectional relationship between 3,655 interactions and stock returns. The Fama-Macbeth envelope regression's envelope dimension is a tuning parameter, which controls the number of repackaged variables the model uses to represent cross-sectional information in the original variables about expected returns. Each row reports results for interaction-based expected return estimates with a given envelope dimension. The sample includes all stocks but microcap stocks.

	Interaction		Characteristic		
Envelope	Slope	t-stat.	Slope	t-stat.	R^2 (%)
1	0.43	3.21	0.28	2.88	1.96
2	0.40	3.75	0.26	2.67	2.00
3	0.39	3.87	0.25	2.63	2.01
4	0.38	4.34	0.24	2.48	1.98
5	0.36	4.91	0.24	2.43	1.94
6	0.34	4.96	0.24	2.49	1.94
7	0.32	5.20	0.25	2.51	1.91
8	0.30	5.42	0.25	2.50	1.90
9	0.29	5.51	0.25	2.53	1.89
10	0.27	5.55	0.26	2.61	1.88
11	0.26	5.75	0.26	2.65	1.86
12	0.24	5.83	0.27	2.68	1.85
13	0.23	5.87	0.27	2.72	1.83
14	0.22	5.80	0.28	2.76	1.83
15	0.21	5.81	0.29	2.84	1.81
16	0.19	5.73	0.29	2.88	1.81
17	0.18	5.68	0.30	2.92	1.80
18	0.17	5.63	0.30	2.95	1.79
19	0.17	5.66	0.30	2.97	1.79
20	0.17	5.67	0.30	2.98	1.79

Table 13: Slopes for Fama-Macbeth regressions of stock returns on out-of-sample expected return estimates using interactions and a range of envelope dimensions. The interaction-based expected return estimates are made with the procedure in section 3.2. The estimation procedure uses the Fama-Macbeth envelope regression model from section 2.3 to estimate the average cross-sectional relationship between 3,655 interactions and stock returns. The Fama-Macbeth envelope regression's envelope dimension is a tuning parameter, which controls the number of repackaged variables the model uses to represent cross-sectional information in the original variables about expected returns. Each row reports results for interaction-based expected return estimates with a given envelope dimension. The sample includes all stocks but microcap stocks.

Envelope	Slope	t-stat.	R^2 (%)
1	0.66	4.02	0.89
2	0.56	4.33	1.02
3	0.53	4.35	1.05
4	0.50	4.74	1.01
5	0.48	5.26	0.94
6	0.44	5.22	0.93
7	0.41	5.39	0.87
8	0.39	5.61	0.83
9	0.37	5.67	0.80
10	0.34	5.71	0.76
11	0.33	5.84	0.72
12	0.31	5.91	0.68
13	0.29	5.96	0.64
14	0.28	5.90	0.62
15	0.26	5.89	0.59
16	0.25	5.84	0.57
17	0.24	5.82	0.55
18	0.23	5.79	0.54
19	0.22	5.82	0.53
20	0.22	5.83	0.52

A Predictor Envelope Dimension Selection

I use a sequential F-test of Osten (1988) to estimate K with degree of freedom estimates from Krämer and Sugiyama (2011). The F-statistic's value for cross-section t is given by

$$F = \frac{PRESS(k) - PRESS(k+1)}{\widehat{DOF}(k+1) - \widehat{DOF}(k)} / \frac{PRESS(m+1)}{N - \widehat{DOF}(k+1)}, \tag{17}$$

$$\widehat{DOF}(m) = \left(\frac{\text{trace}(\Sigma_{X_t})}{\lambda_{\max}}\right) m, \tag{18}$$

where $\lambda_{\max}$ is the largest eigenvalue of Σ_{X_t} and $PRESS(m)$ is the predicted error sum of squares for the PER model estimated with m components. The F-test's null hypothesis is $H_0 : PRESS(m) \leq PRESS(m+1)$, which means increasing the PER envelope dimension from m to $m+1$ does not contain incremental information about covariance between stock returns and firm characteristics. The F-test's alternative hypothesis is $H_1 : PRESS(m) \geq PRESS(m+1)$, which means increasing the PER envelope with dimension m omits some covariance between returns and characteristics necessary for estimating b_t. The F-test is computed sequentially until the null hypothesis is not rejected with a significance level of 0.05.

The F-test above uses degree of freedom estimates from Krämer and Sugiyama (2011). Krämer and Sugiyama (2011) using the PLS regression model's number of components as the model's degrees of freedom is biased downwards and the bias can be large. Specifically, Krämer and Sugiyama (2011) shows if the largest eigenvalue $\lambda_{\max}$ of Σ_{X_t} satisfies

$$\lambda_{\max} \leq \frac{1}{2}\text{trace}(\Sigma_{X_t}), \tag{19}$$

then

$$\widehat{DOF}(1) \geq 1 + \frac{\text{trace}(\Sigma_{X_t})}{\lambda_{\max}}. \tag{20}$$

The condition $\lambda_{\max} \leq \frac{1}{2}\text{trace}(\Sigma_{X_t})$ is true for each characteristic cross-section used later in the paper's empirical results, so the PER model's envelope dimension is certainly a too-small estimate of the model's degrees of freedom when $K = 1$. Additionally, the typical value of $\frac{\text{trace}(\Sigma_{X_t})}{\lambda_{\max}}$ for this paper's empirical results is greater than 20. So, the inequality above indicates the PER model's envelope dimension is typically too-small of an estimate for PER models with envelope dimensions less than or equal to 20 in the paper's sample because the PER model's degrees of freedom when $K = 1$ should be less than the model's degrees of freedom when $K > 1$.

The paper uses the function defined in equation (18) to estimate the degrees of freedom for the PER model with an envelope of dimension m. The degree of freedom estimator defined by equation (18) is an approximation of an unbiased estimator developed by Krämer and Sugiyama (2011) is numerically unstable, i.e. returns a negative degree of freedom estimate, when applied to samples with many covariates and specifically when applied to all of the paper's cross-sections. Equation (18) approximates the unbiased estimator by assuming degrees of freedom are a linear function of $\frac{\text{trace}(\Sigma_{X_t})}{\lambda_{\max}}$ for the PER model. Practically, this approximation produces much larger degree of freedom estimates than the naive approach and empirical results reported by Krämer and Sugiyama (2011) indicate degrees of freedom are roughly proportional to $\frac{\text{trace}(\Sigma_{X_t})}{\lambda_{\max}}$.